MINDSPEED™

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NRZ Bandwidth (-3db HF Cutoff vs SNR) How Much Bandwidth is Enough?

White Paper

Introduction

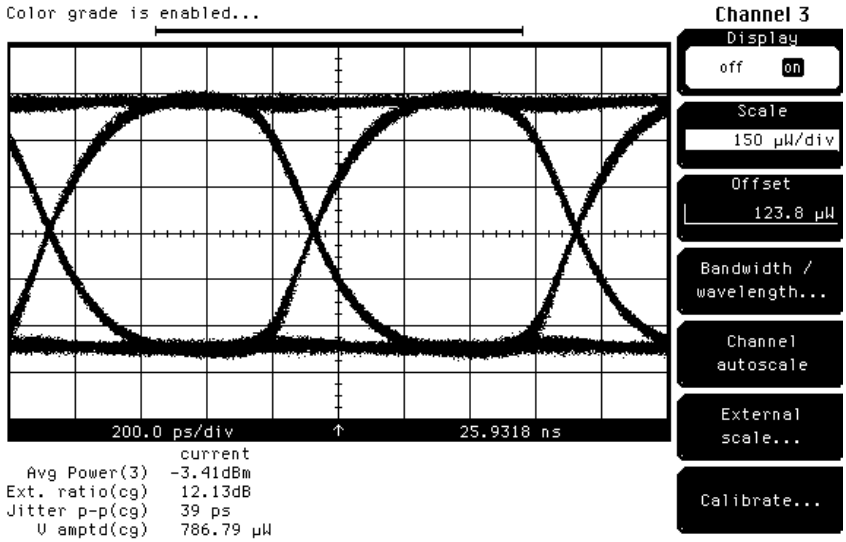
A number of customer-initiated questions have arisen over the determination of the optimum bandwidth for any transimpedance amplifier and subsequent filter employed in a fiber optic receiver module using NRZ coding. When asked what the optimum bandwidth for such a system, most engineers will respond with a number between 0.7 and 0.75 times the NRZ bitrate. The real answer is: It depends on the rolloff rate of the receiver. There is a relationship between rolloff rate and the needed sensitivity, given by the formula in Figure 3. Since fiber-optic receivers are broadband noise limited, their sensitivity is reduced when bandwidth is any greater than absolutely required. The misconception that the "magic number" is between 0.7 and 0.75 times the bitrate comes from industry standards and other manufacturers application notes taken out of context. These standards and application notes either make no mention of the roll-off rate or assume that a single-pole (3db/octave) rate is used. Once the roll-off rate is incorporated, it can be seen that the faster the roll-off rate, the lower the bandwidth required, and the greater sensitivity realized due to the narrower bandwidth over which broadband noise is amplified.

Calculations indicate that a TIA bandwidth of 0.7 to 0.75 times the NRZ bitrate is too high to be optimum, and the reasoning that leads to this number is based on at least two simplifying assumptions. When these assumptions are corrected, the optimal bandwidth is seen to be nearer 0.56 1,2 times the bitrate, which is in agreement with simulation results presented in this paper.

A transimpedance amplifier (TIA) can be designed with excess bandwidth on the assumption that the user will follow it with a precision filter set at the optimum frequency and with the ideal phase response. However precision filters tend to be expensive, with the filter costing significantly more than the TIA itself, let alone the space required to implement the filter. Mindspeed TIAs are designed to incorporate as much of this filtering function internally so as to simplify the external filtering as much as possible, thus producing a design which will have the maximum sensitivity possible at a given data rate.

Frequently Made Assumptions about Optimal Receiver Rolloff Rates

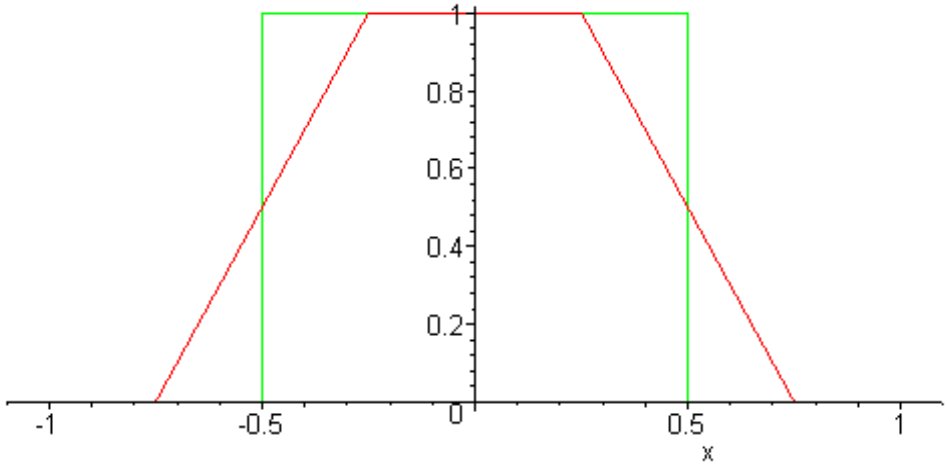
Figure 1. Typical Optical Eye



Two assumptions, frequently made to simplify the math and discussion are that the input signal have 0 ps rise/fall times and ignoring the effects of amplifying broadband noise. These assumptions are examined below.

The first assumption is that the input signal to the TIA is an ideal rectangular wave pulse stream with vertical sides. Apart from the fact that this is impossible, it is also a poor approximation of the real world. If the input signal is a stream of trapezoidal pulses a closer representation to reality appears, even though this is still an approximation. Since the trapezoidal pulse stream has less energy at higher frequencies, in an argument based on the power spectrum of the bit stream, a different measurement will result. Figure 1 shows the measured optical output eye from a typical fiber optic laser and driver combination. Approximating this with a piece-wise-linear model for simplicity, the unit pulse has 10% to 90% rise and fall times that are 20% of the period. This is a reasonable assumption for the typical case. Even if the signal starts out with more vertical rise and fall times, as is the case when the signal has a great distance over a fiber (and therefore where the TIA sensitivity is critical) various dispersion mechanisms are likely to mean that the rise and fall times are broadened. Figure 2 compares the trapezoidal unit pulse with the rectangular unit pulse used in simplified examples. Because of discontinuities in the trapezoidal waveform, it is likely to have more high frequency content than the waveform in Figure 1 that is being approximated.

Figure 2. Trapezoidal and Rectangular Unit Pulses



The second assumption is that the system noise is constant over the entire frequency spectrum. In any practical TIA the thermal noise has two components, a constant with frequency term whose source is primarily the transimpedance setting feedback resistor, and a term proportional to the frequency squared, whose source is primarily the active input devices (see Figure 3)[5]. This second term dominates the overall noise. When these terms are integrated over the bandwidth, the constant term contributes proportionally to bandwidth and the second term proportionally to bandwidth cubed. As a result the effect on the signal to noise ratio of having excess bandwidth is more marked that suggested by more simplified examples which only use the constant term.

Figure 3. Theoretical expression for TIA thermal noise, assuming input transistors and feedback resistor are the dominant noise sources.

$$noisesq := 4\,\frac{K\,T\;1\;\frac{2}{3}\,\frac{1}{(Gmn\quad Gmp)\,Rf}}{Rf}\;\frac{8}{3}\,\frac{K\,T\;{}^{2}\,(Cgn\quad Cgp\quad Cin)^{2}}{Gmn\quad Gmp}$$

Autocorrelation of Random NRZ Data

By extracting the autocorrelation function of the unit pulse and then taking the Fourier transform of the result the power spectrum of random NRZ data can be found. However, the autocorrelation function and power spectrum will be different if the unit pulse is assumed to be trapezoidal and not rectangular. Figure 4 shows the autocorrelation functions of the rectangular and trapezoidal random NRZ pulse trains together. The autocorrelation function of the rectangular pulse stream is an equilateral triangle, however, the autocorrelation function of the trapezoidal pulse stream has an appearance roughly similar to a Gaussian function. Since the changes in slope of the Gaussian distribution are not as abrupt as the triangle, it should follow that the Fourier transform would have fewer terms at higher frequencies than the triangle with its discontinuities (abrupt changes in slope).

Figure 4. Autocorrelation Function of Trapezoidal and Rectangular Random NRZ Bitstreams

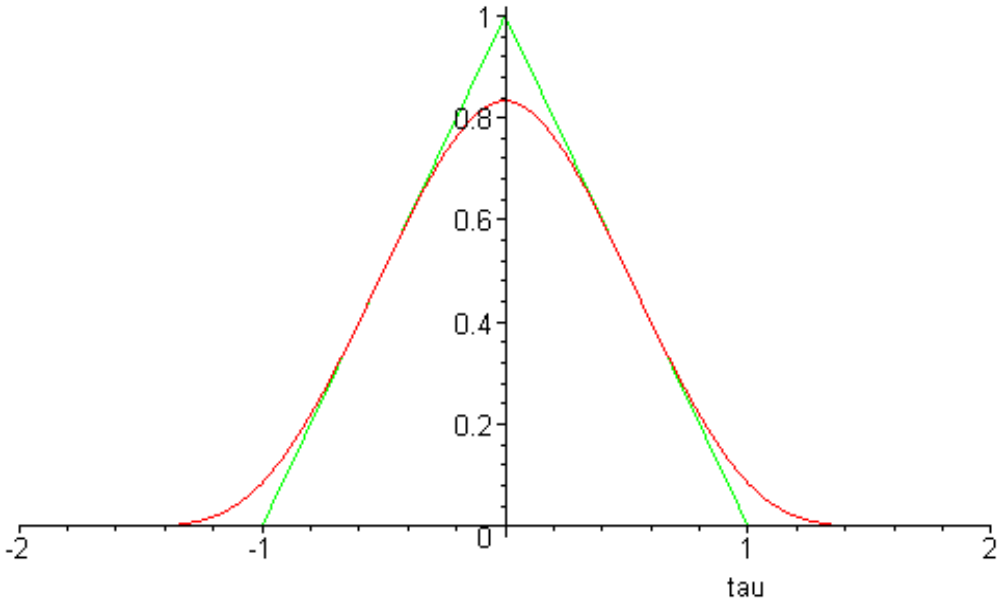
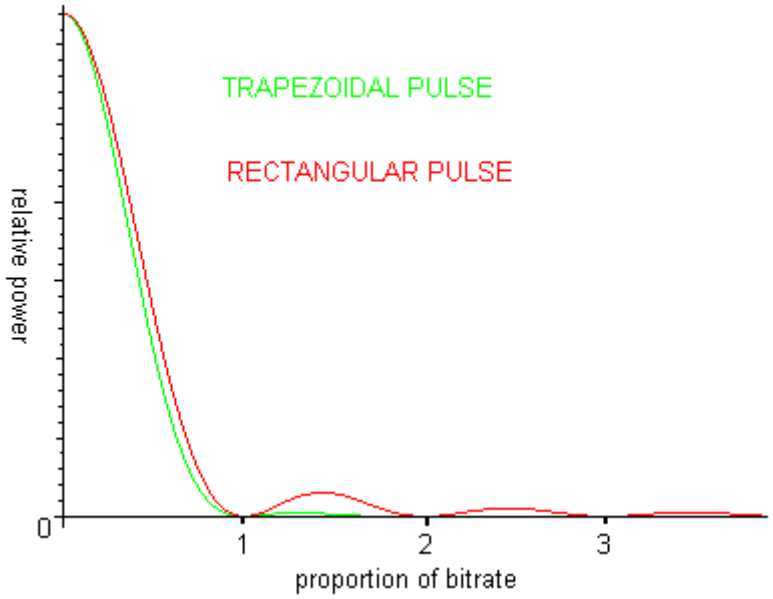


Figure 5. Power Spectra of Random NRZ Data

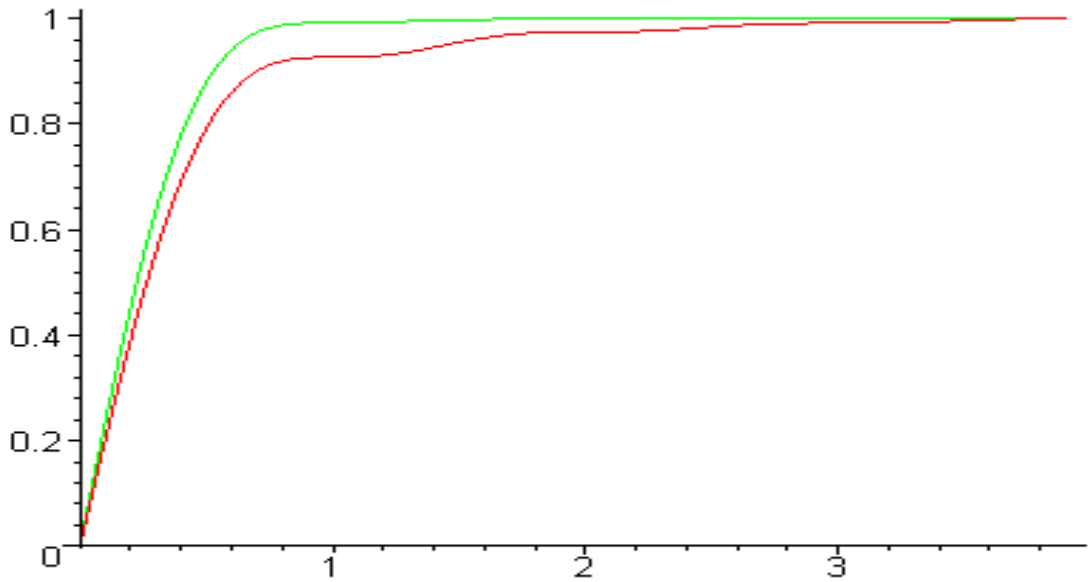


Power Spectrum of Random NRZ Data

The power spectrum of the NRZ random data stream can be found by taking the Fourier transform of the autocorrelation function. Figure 5 compares the power spectra of the trapezoidal and rectangular random NRZ data streams. Although the differences appear small, it can be seen the trapezoidal pulse stream has less power at higher frequencies, and is enough to change the optimum bandwidth significantly.

Figure 6 shows the cumulative power spectra for both waveform approximations. If, as suggested by some sources 93% of the power spectrum 'in band' is sufficient, then the necessary bandwidth is reduced to about 0.6 times the bitrate. However, if the signal to noise ratio is taken into account, it can be seen that the result is affected by the system bandwidth. Here the system noise is accounted for, in the shape of the noise power vs. frequency curve. It has also been found that the exact shape of the amplitude vs. frequency rolloff curve may also affect the signal to noise ratio.

Figure 6. Cumulative (Integrated) Power Spectrum of Random NRZ Data



If the TIA bandwidth is chosen where the signal to noise ratio is optimal, and this is applied to Figure 6, it appears that a bandwidth of 0.75 times the bitrate is a reasonable figure for both the trapezoidal (red) and rectangular (green) pulse streams. For both curves, it appears that the flat section to the right of 0.75 times the bitrate and the steeply falling section to the left make this a good choice, however, this does not account for the fact that the dominant noise source contributes a power that is proportional to the bandwidth cubed. For instance, Figure 7 shows the cumulative integrated input referred noise vs. bandwidth (as a proportion of bitrate) for a Mindspeed TIA. This curve, derived from multiple AC noise simulations, can be modelled with a third order polynomial to a high accuracy (correlation coefficient=0.999), and is typical of the Mindspeed TIAs. If the data taken from Figure 6 is divided by the data taken from Figure 7 and is arbitrarily normalized to a signal to noise ratio of 1, to find how the signal to noise ratio varies with bandwidth using a trapezoidal pulse, with a BW/BR=0.5, the curves in Figure 8 result. Here, the result is that the signal to noise ratio actually peaks when the filtered bandwidth is only about one quarter of the bitrate. This result is not immediately obvious and it could be

expected that a TIA filtered down to a quarter of the bitrate to show a higher BER than the same TIA filtered down to one half the bitrate. However, there are cases of TIAs with apparently close to half the bit rate for –3dB electrical bandwidth that still appear to have good sensitivity. Figure 7 shows that, with a 'brick wall' filtered output, changing the filter bandwidth from 0.75 times the bitrate to 0.5 times the bitrate would result in a 50% improvement in signal to noise ratio. This translates into about a 1.7dBm apparent improvement in the optical sensitivity. The missing part of this equation is the effect of intersymbol interference (ISI). When the bandwidth decreases to less than half the bitrate, the individual rapidly. This closure, when added to the thermal noise, makes it statistically more likely that the post amplifier following the TIA will output the wrong data and the BER will increase. Unlike thermal noise, this is a systematic form of eye closure, which will be repeatable for the same bit sequence. It is possible to remove this by examining the bit sequence and shifting the slicing level dynamically by an appropriate amount, and slicers exist that perform this function. As a result, ISI is a pulses begin to merge and the eye closure increases more benign form of noise than thermal noise. ISI can be seen in transient simulations, but not in AC noise simulations.

Figure 7. Simulated Noise, with Signal to Noise vs. Bandwidth Normalized

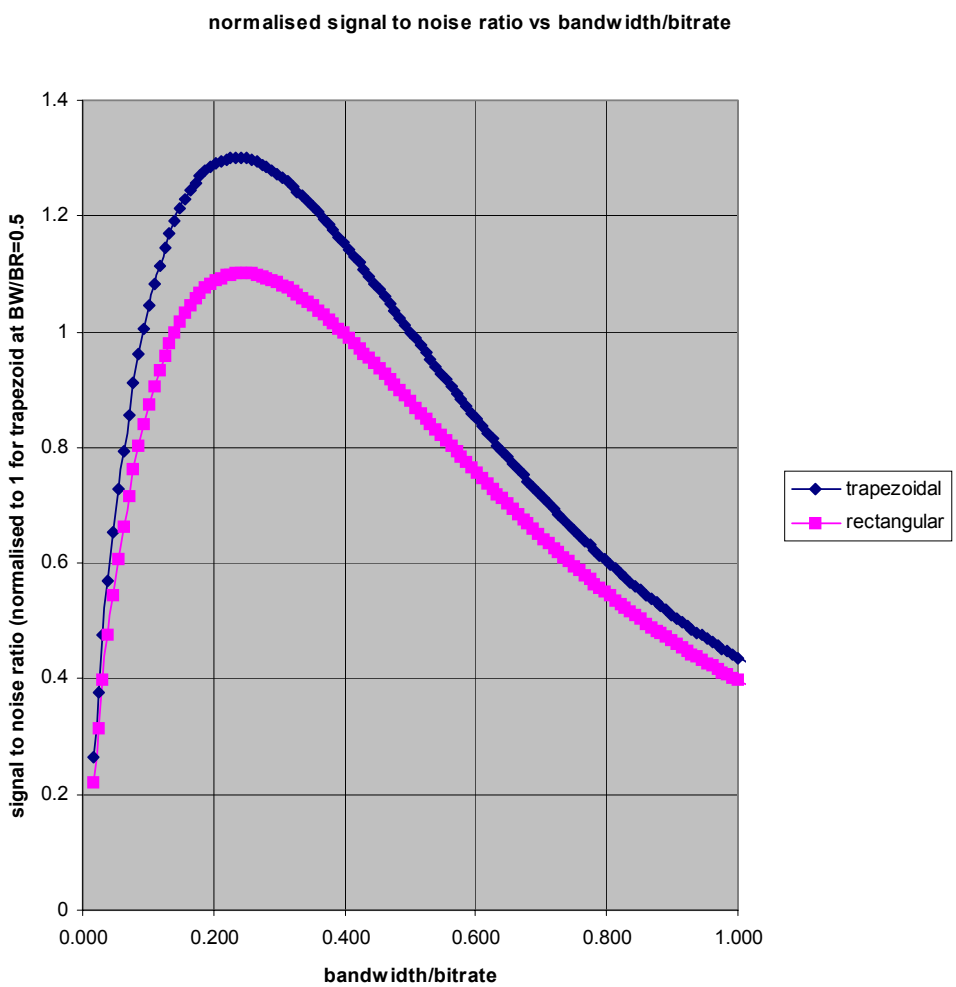


Figure 8. Typical Input Referred Noise vs. Bandwidth

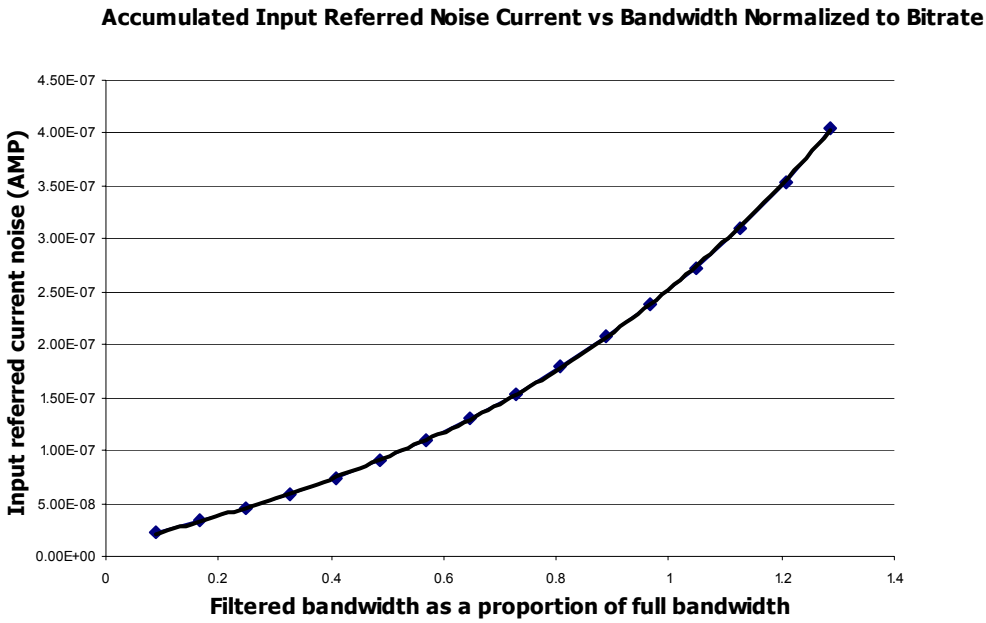


Figure 9 shows the transient eye outputs for a typical Mindspeed TIA, when the bit period is shifted from 0.5´nominal to 2´nominal with the time scale normalized to half the bitrate. The plot in the second column of the second row shows the simulated ISI. At this point the vertical eye closure is just beginning to get significant. Figure 10 is generated from the Figure 9 plots. It shows the eye height divided by the projected vertical eye closure (i.e. S/N ratio) plotted against the –3dB bandwidth. It can be seen from this plot that once the bandwidth gets to 0.75 times the bitrate no further reduction of noise due to vertical eye closure (by increasing the bandwidth) is possible, and this may be seen as supporting the 0.75 figure frequently cited, but does not account for the fact that the sensitivity may be improved by trading some thermal noise for ISI noise. Thermal noise is fixed, however ISI is directly proportional to signal level, i.e. it is the signal to noise ratio which is fixed. The designer's task is to pick the bandwidth, which gives the optimum mix of ISI noise and thermal noise to maximize sensitivity. To make this realistic, the real roll off shape of the TIA bode plot must be accounted for, and the brick wall filter response example discarded.

In order to determine the effect of bandwidth with a realistic roll off on the thermal noise, the gain vs. frequency bode plot is shifted with respect to the frequency of interest. By multiplying the input referred noise by the gain in each band; integrating over the bandwidth and then taking the square root, the residual thermal noise vs. –3dB bandwidth is derived. The result is Figure 11, which shows the integrated output thermal noise vs. –3dB bandwidth. Figures 10 and 11 show that for a given output signal level, the proportion of thermal plus ISI noise in the signal as a function of the –3dB bandwidth. Figure 12 shows the results of assuming an output level of 20 V. This clearly shows that the optimum –3dB electrical bandwidth is about 0.56 times the bitrate, which agrees with a 1991 paper written by Gareth Williams[4]. Williams also references previous works by Smith and Personick, which At lower bandwidth the noise is dominated by ISI and at higher bandwidth by thermal noise.

Figure 9. Output Eye Diagrams with Bit Period From 0.5 to 2x the Nominal in 0.1 Steps

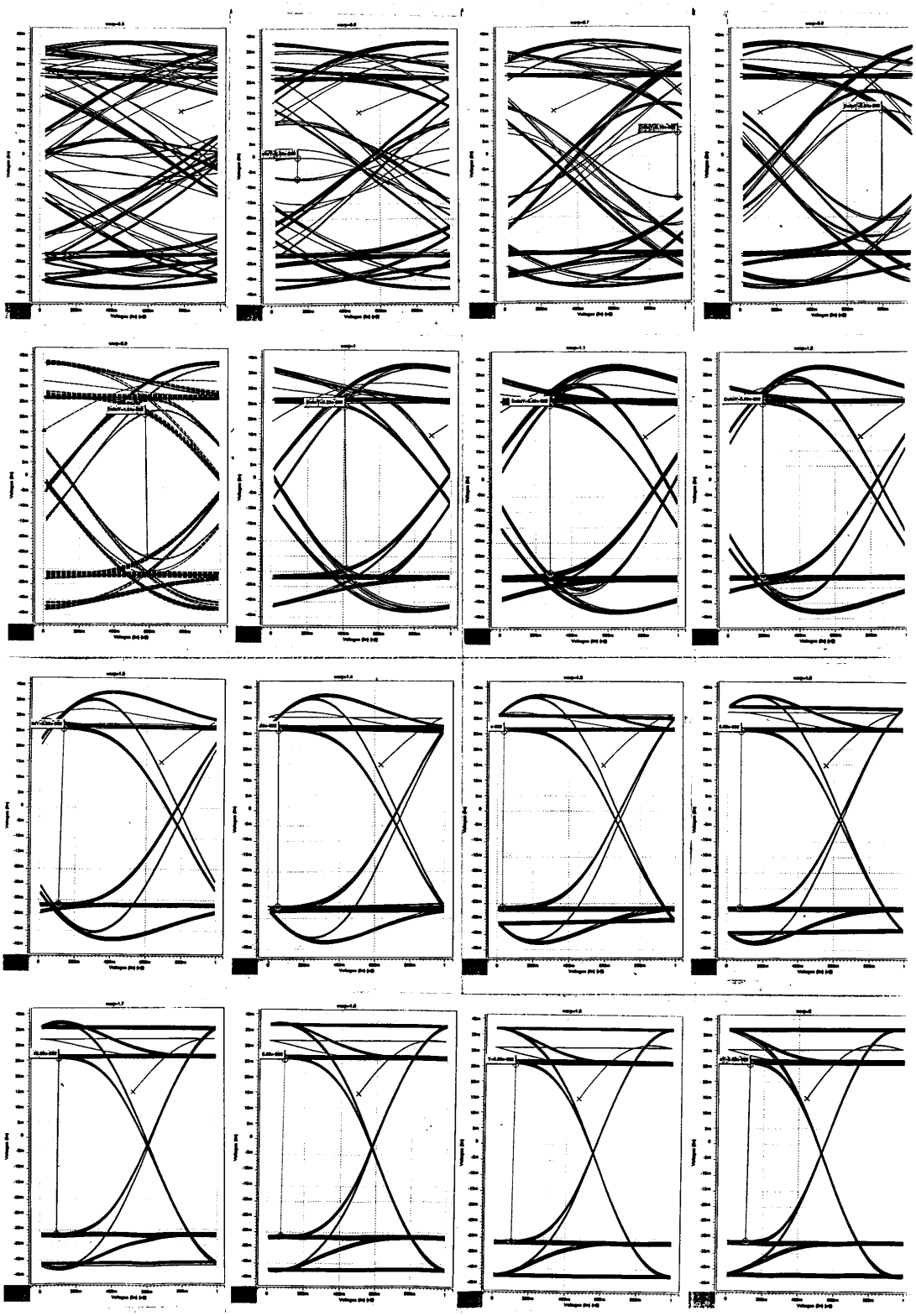


Figure 10. Effect of Bandwidth on ISI Noise

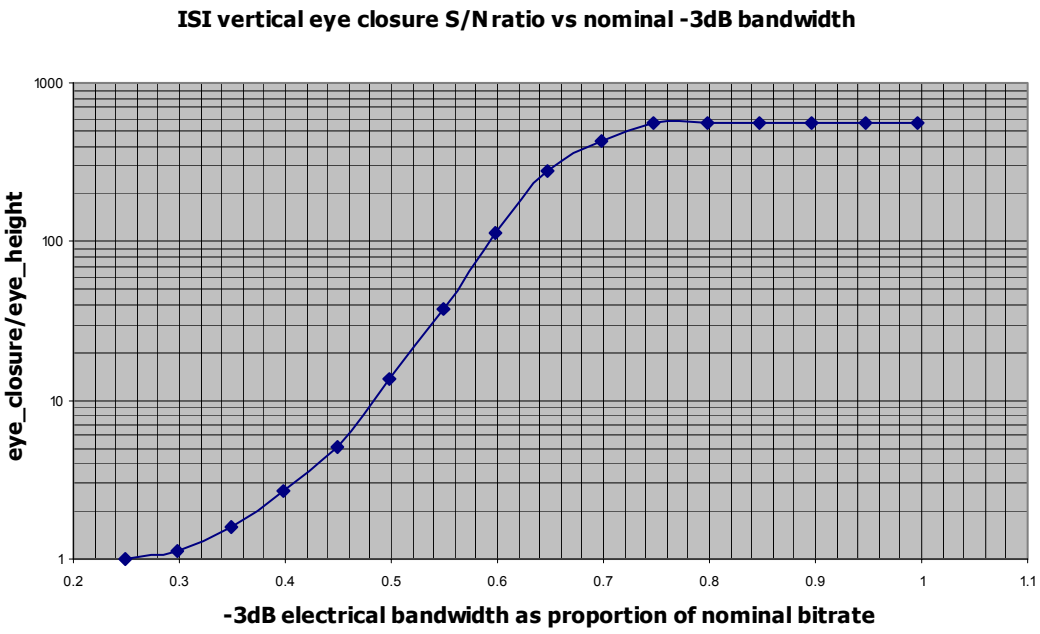


Figure 11. Output Noise vs. -3dB Bandwidth

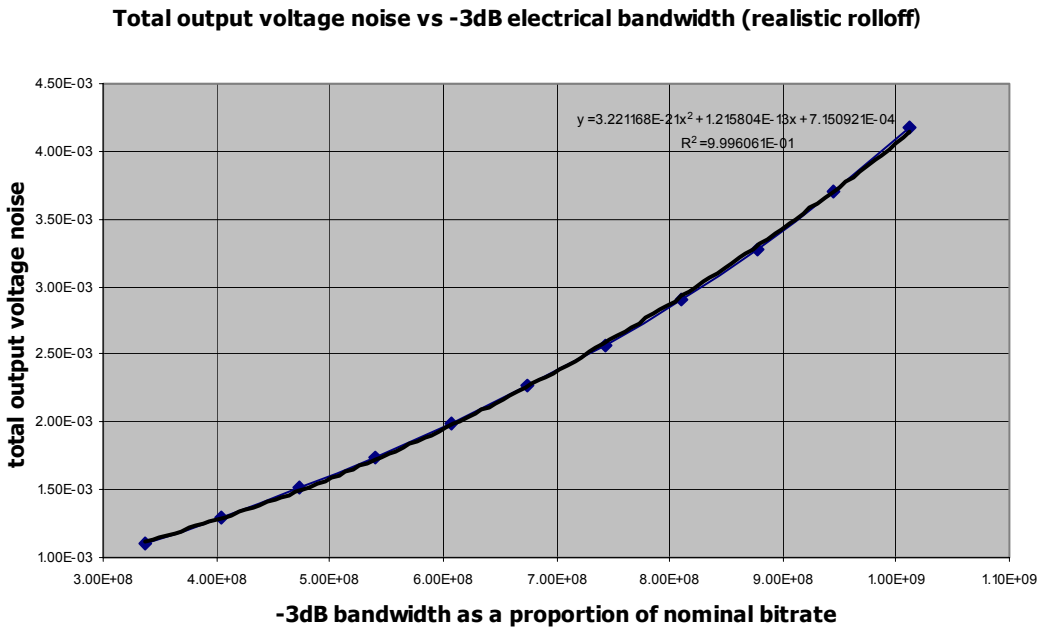
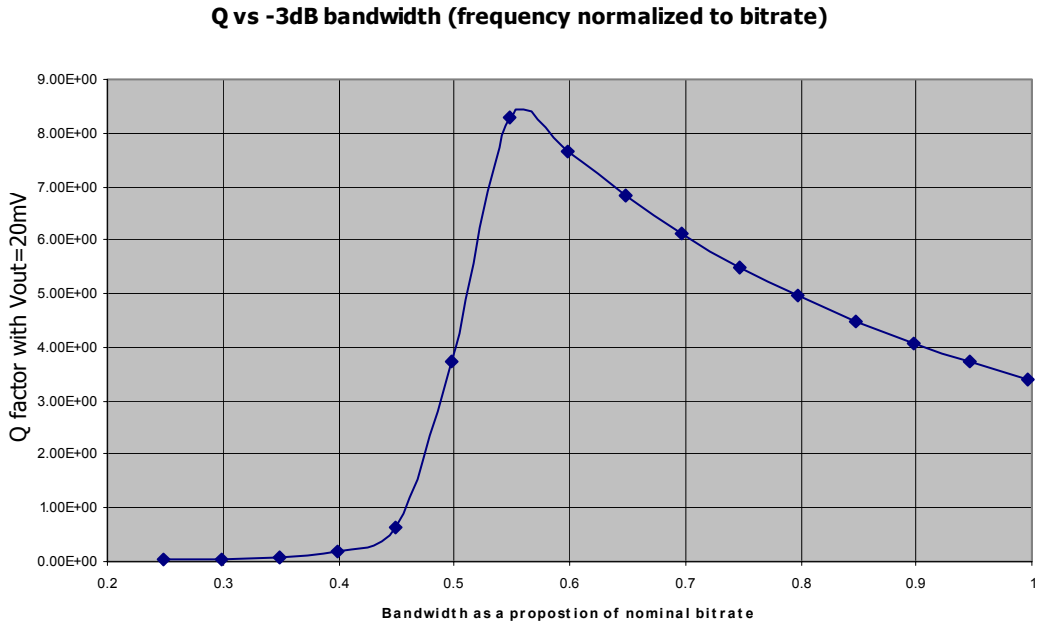


Figure 12. Q factor (Signal to Noise Ratio) vs. -3dB Bandwidth for 20 mV Output Signal



To prove this method with actual optical assemblies, 1.25 Gb optical subassemblies were tested under the following characteristics were tested: Input power of –28dBm, extinction ratio of 10dB, photo-diode responsivity of 0.9A/W. An output of 20 mV, giving a mid-band gain of 18 KW in theory would require a theoretical sensitivity of –28.2 dBm. Measured parts had a BER better than 1e-10 with the optical input as given above.

At the nominal (simulated) TIA bandwidth of 0.64 x (nominal bitrate), Figure 12 shows a Q of between 6.4 and 7 corresponding to a BER of between 1e-10 and 1e-12. As a comparison, actual devices made using the a Mindspeed 1.25 Gb TIA, the best sensitivity measured at room temperature was –28.1 dBm, and all of the devices had a sensitivity of -27 dBm or better.

Conclusion

- 0.7 to 0.75 times the Bitrate for optical receiver bandwidth is probably optimal if the rolloff rate for the TIA is –3db per Octave.
- Mindspeed TIAs do not have a –3 db/Oct rolloff rate. The rolloff rate for Mindspeed TIAs is –9 db/Octave, resulting in greater sensitivity since less broadband noise is amplified.
- 0.56 times the Bitrate for a –9 db/Octave rolloff rate is the minimum bandwidth required for a receiver with –9 db/Oct rolloff rate.
- The noise amplified over a lower bandwidth with a –9 db/Oct rolloff rate, yields a PIN/TIA receiver which is ~2 db more sensitive than one which has a –3 db/Oct rolloff rate, for a given BER.
- If the eye shape is too rise/fall time limited, and sensitivity is not an overriding consideration, select the Mindspeed TIA one speed grade higher.

References

1) Personick, S.D. Bell System Technical Journal # 52, pp. 843; 1973

2) Personick, S.D. Bell System Technical Journal # 52, pp. 875; 1973

3) Smith, R.G and Personick, S.D "Semiconductor Devices for Optical Communications" Chapter 4, Springer-Verlag; 1980

4) Williams, G.F Bell System Technical Journal pp 105 (1983)